Donc $\boxed{J = 2\ln 2 - \frac{1}{2}}$

Exercice 9

$f(x) = \dfrac{1+\ln x}{x}$; $x > 0$

1) f est dérivable sur $]0,+\infty[$ et

$$f'(x) = \frac{(1+\ln x)'x - (x)'(1+\ln x)}{x^2}$$

$$= \frac{\left(\frac{1}{x}\right)\times x - 1 - \ln x}{x^2} = \frac{1 - 1 - \ln x}{x^2}$$

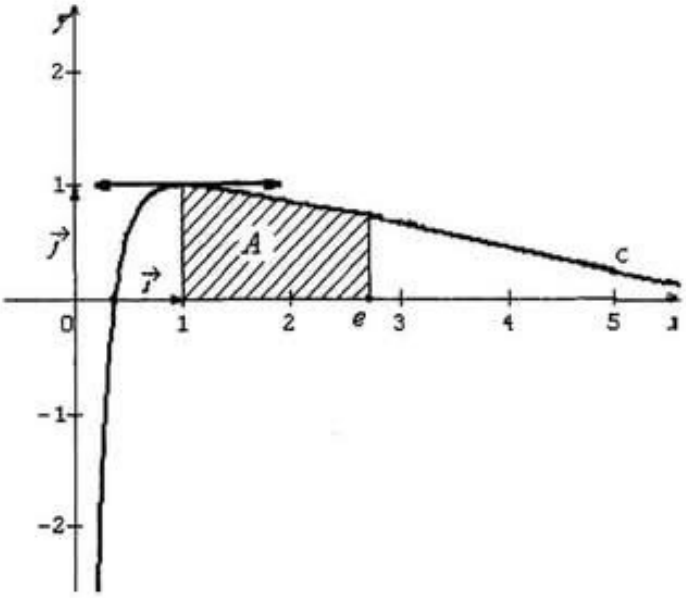
Le signe de $f'(x)$ est celui de $(-\ln x)$ car $x^2 > 0$

x	0		1		$+\infty$
$f(x)$	‖	+	0	−	
$f(x)$	‖ $-\infty$	↗	1	↘	0

$$\lim_{x\to 0^+} \frac{1+\ln x}{x} = \lim_{x\to 0^+} \frac{1}{x}(1+\ln x) = -\infty$$

$$\lim_{x\to +\infty} \frac{1+\ln x}{x} = \lim_{x\to +\infty} \frac{1}{x} + \frac{\ln x}{x} = 0$$

$$f(1) = \frac{1+0}{1} = 1$$



2)

$$A = \int_1^e |f(x)|dx = \left(\int_1^e f(x)\,dx\right)\|\vec{i}\|.\|\vec{j}\|$$

$$= \int_1^e \frac{1+\ln x}{x}dx = \int_1^e \left(\frac{1}{x} + \frac{1}{x}\ln x\right)dx$$

$$= \int_1^e \frac{1}{x}dx + \int_1^e (\ln x)'.\ln x\,dx$$

$$= [\ln x]_1^e + \left[\frac{1}{2}(\ln x)^2\right]_1^e$$

$$= \ln e - \ln 1 + \frac{1}{2}(\ln e)^2 - \frac{1}{2}(\ln 1)^2$$

$$= 1 + \frac{1}{2} = \frac{3}{2}\|\vec{i}\|.\|\vec{j}\|$$

Exercice 10

$f(x) = x\ln(x+2)$, $x \in [0,+\infty[$

1) $f'(x) = \ln(x+2) + x\times\dfrac{1}{x+2}$

Comme $x \geq 0$ donc $f'(x) > 0$

$\lim\limits_{x\to+\infty} f(x) = \lim\limits_{x\to+\infty} x\ln(x+2) = +\infty$; $f(0) = 0$

x	0		$+\infty$
$f(x)$		+	
$f(x)$	0	↗	$+\infty$

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